COLOR IMAGE CODING USING 4D NTH-ORDER WALSH ORTHOGONAL TRANSFORM

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ABSTRACT

The concept of 2D matrix is extended to 4D matrix using multi dimensional matrix theory. 4D n^{th} order Walsh orthogonal transform is formed and is applied on image along with cosine transform. The model of 4-D n^{th} order matrix can process 4-D data in a unified mathematical model. It has high efficiency of classical matrix transform in the aspect of removing redundancy of color space. So this transform found effective than jpeg standards in color image coding there by compression.

Keywords: Multidimensional matrix, 4D n^{th} order Walsh transform, orthogonal matrix, coding of coefficients, compression.

I. INTRODUCTION

The rapid changes in technology make the information exchange to involve images and video. So for huge amounts of data storage and transmission compression and coding is necessary. In this paper one of the efficient ways of compression using Walsh transform is introduced. In order to reduce the redundancy of color space, 2-D matrix is extended to 4-D matrix. The three frames in color image are processed in a unified way. Better compression results are obtained.

II. DEFINITION AND OPERATION LAWS:

A. Definition of 4D Matrix

If not specified in this paper, R is supposed to the set of real number, C is supposed to the set of complex number and H is supposed to the set of quaternion,

$$F \in \{R; C; H\}$$

Definition 1: The definition of 4D matrix

The 4D arrangement $[a_{ijst}I \times J \times S \times T$ on F is called $I \times J \times S \times T$ order 4D matrix.

Definition 2: The definition of 4D nth-order matrix

For any $I \times J \times S \times T$ order 4D matrix A, if it's order meets: $I = J = S = T = n$, that is $A_{I \times J \times S \times T}[a_{ijst}]_{I \times J \times S \times T}$, call the A is 4D nth-order matrix. Denoted as $A_{IV,\,n} = [a_{ijst}]_{IV,\,n}$ Obviously, 4D nth-order matrix is 4Dnth-order square matrix, is a special class of 4D matrices.

B. Operation Laws

- The equality law

For any 4D nth-order matrix, $A_{IV,\,n} = [a_{ijst}]_{IV,\,n}$, $B_{IV,\,n} = [b_{ijst}]_{IV,\,n}$ if their corresponding elements are equal, namely:

$$a_{ijst} = b_{ijst}$$
$$(1 \le i \le n, 1 \le j \le n, 1 \le s \le n, 1 \le t \le n)$$

Then 4D nth-order matrix A is said to be equal to B, which denoted as $A = B$.

- The addition law

Suppose 4D nth-order matrix $A_{IV,\,n} = [a_{ijst_{IV,\,n}}$ and $B_{IV,\,n} = [b_{ijst}]_{IV,\,n}$ the addition law between matrices is:

$$C_{IV,\,n} = [c_{ijst}]_{IV,\,n} = [a_{ijst} + b_{ijst}]_{IV,\,n}$$

We call the above formula the sum of A and B, denoted as $C = A + B$.

- The multiplication of 4D nth-order matrix

Generally, suppose 4D nth-order matrix $A_{IV,} = [a_{ijst_{IV,\,n}}$ $B_{IV,\,n} = [b_{ijst}]_{IV,\,n}$, $C_{IV,\,n} = [c_{ijst}]_{IV,\,n}$ so the definition of the multiplication of 4D nth-order matrices is:

$c_{ijst} = \sum_{p}^{n} \sum_{q}^{n} = 1\; aijst\, bijst$ Can be denoted as $C = AB$.

- 4D nth-order identity matrix

4D nth-order matrix is $A_{IV,\, n} = [a_{ijst_{IV,\, n}}$, if

$$a_{ijst} = \left\{ \begin{array}{ll} 1 & (ij) = (s, t) \\ 0 & (ij) \neq (s, t) \end{array} \right\}$$

$(1 \le i \le n;\ 1 \le j \le n;\ 1 \le s \le n;\ 1 \le t \le n)$

$A_{IV,\, n}$ is called 4D nth-order identity matrix.

- Transpose of 4D nth-order matrix

Suppose 4D nth-order matrices

$A_{IV,\, n} = [a_{ijst}]_{IV,\, n}$ and $B_{IV,\, n} = [b_{ijst}]_{IV,\, n}$

If

$b_{ijst} = a_{stij}(1 \le i \le n;\ 1 \le j \le n;\ 1 \le s \le n;\ 1 \le t \le n)$, is called the transpose matrix of $A_{IV,\, n}$.

- Kronecker product of 4D nth-order matrices

Suppose

$A\; I_1 \times I_2 \times I_3 \times I_4 = (a_{ijst})\; I_1 \times I_2 \times I_3 \times I_4$

$B\; J_1 \times J_2 \times J_3 \times J_4 = (b_{ijst})\; J_1 \times J_2 \times J_3 \times J_4$

So the block matrix:

$A \otimes B = (a_{ijst}\, B)\; (I_1 \times J_1) \times (I_2 \times J_2) \times (I_3 \times J_3) \times (I_4 \times J_4)$ Is called kronecker product on A and B.

III. 4D NTH-ORDER WALSH ORTHOGONAL TRANSFORM

On the basis of the multiplication of 4D nth-order vector matrices, Walsh transform theory is introduced. We find the 4D orthogonal transform operator based on the

4D vector matrix, and apply the operator to color image compression coding. The transform kernel is:

$$w\,(x, u, y, v, z, p, g, q) = \frac{1}{\sqrt{MNRS}}$$

$$(-1) \sum_{i=0}^{m-1} bi\,(x)\; bm - i - 1\; (u) +$$

$$\sum_{i=0}^{m-1} bi\,(x) - i - 1\; (u) +$$

$$\sum_{i=0}^{m-1} bi\,(x)\; bm - i - 1\; (u) +$$

$$\sum_{i=0}^{m-1} bi\,(x)\; bm - i - 1\; (u)$$

4d Walsh transform is:

$$F\,(u, v, p, q) =$$

$$\sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \sum_{Z=0}^{R-1} \sum_{g=0}^{s-1} f\,(x, y, z, g)$$

$$w\,(x, u, y, v, z, p, g, q)$$

$(u = 0, 1, 2 \ldots, M - 1\;\; y = 0, 1, 2 \ldots N - 1$

$z = 0, 1, 2 \ldots R - 1\;\; g = 0, 1, 2 \ldots S - 1)$

The 4D Walsh transform is written in matrix form, that is:

$$F = W_{IV}\, f\, W_{IV}$$

In which W_{IV} is 4D kernel matrix

A. 4D 2nd Order Walsh Orthogonal Transform Matrix

According to 4D Walsh transform formula, we can write 4D Walsh kernel matrix. 4D 2nd order Walsh

Transform matrix is as follows:

$W_{IV,2} =$

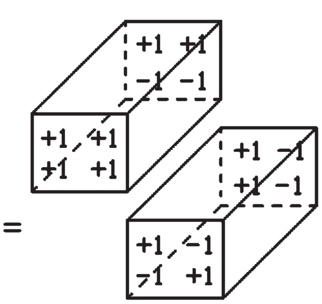


Fig. 1. 4D 2nd order Walsh transform matrix

According to the 4D nth-order matrix operation laws, $W_{IV,2}$ is symmetric matrix, an is orthogonal matrix. So the multiplication between $W_{IV,2}$ and itself is identity matrix.

We can get the conclusions as follows:

- The matrix consists of 1 and – 1.
- The sequence order is increasing.

Form Fig. 1 we can see, the sequence order of each surface of 4D 2nd order Walsh vector matrix is increasing, that is the first is 1, the second is 2, and so on. This property is analogous to frequency increasing property.

- This matrix is orthogonal.

B. 4D $\mathbf{2^K}$ *Order Walsh Matrix*

$$W_{IV,2}^{k} = \begin{bmatrix} W_{IV,2}^{k-1} & W_{IV,2}^{k-1} \\ W_{IV,2}^{k-1} & W_{IV,2}^{k-1} \end{bmatrix}$$

$$= W_{IV,2} \otimes W_{IV,2}^{k-1}$$

in which $2 \le K \in N$

C. 4D Nth-order Walsh Transform Matrix

4D nth-order matrix direct transform:

$$B_{III,n} = W_{IV,n} A_{III,n} C_{III,n}$$

4D nth-order matrix inverse transform:

$$A_{III,n} = W_{IV,n} B_{III,n} C_{III,n}$$

In which, $A_{III,n}$ is the three-dimensional sub block of the image sequence which to be transformed.

$C_{III,n}$ is the two-dimensional discrete cosine orthogonal transform matrix under the usual sense. $W_{IV,n}$ is the 4D nth-order.

Orthogonal transform matrix in our paper.

B. Color Image Transform Coding

Step1: Divide the actual image in to $8 \times 8 \times 3$ sub blocks.

Step2: 4d 8th order Walsh orthogonal transform is deduced.

Step3: Walsh matrix is applied on $8 \times 8 \times 3$ sub blocks of R G B planes.

Step4: On the resultant matrix Cosine transform is applied.

Step5: The sub blocks are merged and divided into R G B planes.

Step6: Thresholding is applied on coefficients.

Step7: Runlength and Huffman coding are applied.

Step8: Compression ratios are calculated with different thresholding values.

Step9: After that the coefficients are decoded.

Step10: Inverse Cosine transform and Walsh transform are applied.

Step11: PSNR values are calculated for R G B planes

Step12: Planes are concatenated and retrieved image is shown

Step13: All the steps from 1 to 12 are repeated for 2d Walsh transform and is compared with 4d.

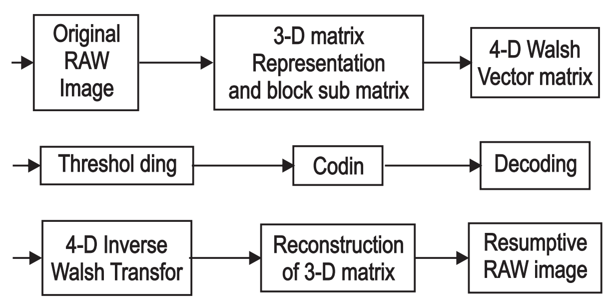


Fig. 4. Three-dimensional matrix representation of the color image

First we introduce the three-dimensional representation of color image in the initial condition. The size of a color image is I × J and composed of R, G and B planes of the same size. So according to the characteristics of the color image, three-dimensional matrix representation is given as in Fig. 4.

The mathematical expression is:

$$A\, I \times J \times 3 = [a_{ijk}]_{I \times J \times 3}$$

$$\begin{bmatrix} R_{11} & R_{22} & \cdots & R_{1J} \\ R_{21} & R_{22} & \cdots & R_{2J} \\ R_{11} & R_{12} & \cdots & R_{1J} \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{IJ} \\ G_{21} & G_{22} & \cdots & G_{2J} \\ G_{11} & G_{12} & \cdots & G_{IJ} \end{bmatrix}$$

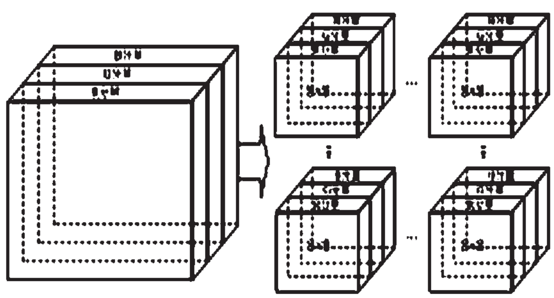
$$\begin{bmatrix} B_{11} & B_{22} & \cdots & B_{IJ} \\ B_{21} & B_{22} & \cdots & B_{2J} \\ B_{11} & B_{22} & \cdots & B_{JJ} \end{bmatrix}$$

Where $(1 \le i \le I, 1 \le j \le J)$ denote the spatial position of color image pixel. $I \le k \le 3$ is the three components of color image, represented by three vertical plane vectors of three-dimensional matrix.

The blocked mode divides the RGB matrix into $8 \times 8 \times 3$ sub matrices, which is called RGB blocked mode. The schematic diagram is:

Fig. 5 RGB blocked mode

The color images in our experiment are color



images from the library of test, pepper, leena and baboon. The transforms applied are as given below

$$F_{\overline{8 \times 8 \times 3}} = W_{\overline{8 \times 8}} \times_{\overline{8 \times 8}} f_{\overline{8 \times 8 \times 3}}\, C_{\overline{8 \times 8}}$$

Here $f_{8 \times 8 \times 3}$ is the sub block of original color image, $F_{8 \times 8 \times 3}$ is the matrix after transform, $C_{8 \times 8}$ is two-dimensional discrete cosine transform operator. Then the transform coefficients are thresholded and encoded.

IV. EVALUATION STANDARDS AND EXPERIMENTAL RESULTS

A. *Evaluation Standard*

We consider two parameters: compression ratio (CR) and peak signal to noise ratio (PSNR) as the evaluation standards in the results of our experimental.

CR is the ratio of original image and the image after orthogonal transform. The formula is:

$$\text{CR} = \frac{256 \times 256 \times 8 \times 3}{b_{po}}$$

In which b_{po} are the bits of the image after

Transformation.

The peak signal to noise ratio evaluates the quality of image objectively, that is the similarity between reconstructed image and original image. The formula is:

$$\text{PSNR} = 10 \lg \frac{255^2 \text{MN}}{\sum_{m=1}^{M} \sum_{n=1}^{N} [f(m,n) - g(m,n)]^2}$$

$$= 10 \lg \frac{255^2}{\text{MSE}}$$

Here MSE means mean square error of reconstructed image.

For color image the reconstructed image has RGB planes. So the average of peak signal to noise ratio is

$$\text{PSNR} \; \frac{\text{PSNR}_R + \text{PSNR}_G + \text{PSNR}_B}{3}$$

If PSNRs are equal, the better the CR is the better the algorithm is.

In the below table different CR and PSNR values for 2d and 4d walsh transforms are compared.

Fig. 6. Images with different compression ratios

Table 1 Comparison Results of 4D 8^{th} Order Walsh Transform and JPEG

Walsh transform			CR		PSNR		
				PSNRr	PSNRg	PSNRb	PSNR Avg
Pepper. png	Jpeg		20.93	40.14	36.26	40.58	38.99
	Data in paper	2d	14.53	25.04	26.05	26.12	25.73
			51.19	28.44	21.59	21.28	23.77
		4d	35.67	32.42	30.29	28.42	30.37
			27.41	37.76	37.66	37.04	37.48

Walsh			CR		PSNR		
				PSNRr	PSNRg	PSNRb	PSNRavg
Leena. png	jpeg		14.03	28.91	33.83	33.84	32.19
	Data in paper	2d	12.64	25.86	24.86	25.78	25.50
		4d	34.09	27.92	21.39	24.45	24.59
			29.44	29.61	27.69	29.33	28.88
			26.27	31.97	31.34	31.57	31.63
Baboon. bmp	jpeg		13.73	37.22	28.00	29.90	31.71
	Data in paper	2d	12.54	24.86	24.84	23.90	24.53
		4d	32.29	25.69	25.90	22.05	24.55
			30.39	28.56	28.34	26.63	27.84
			27.08	31.21	31.17	30.94	31.10

V. CONCLUSION

We introduced 4d matrix concept, based on that we deduced 4d Walsh orthogonal transform which is extended to nth order. R G B planes are separated and processesed in a unified way. Favourable compression results are obtained

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